Math 45 1.7 Exponents and the Order of Operations

1. **Review exponents: notation, signs**
2. **Use calculator: different calculators work differently, learn how yours works**
3. **Use order of operations: by hand and with calculator**

The order of operations is a list of rules about the order we do the parts of a calculation containing several parts. Calculators have been programmed to follow the order of operations.

Step 1: Identify all grouping symbols and resolve them from the inside out. Grouping symbols include purely grouping symbols and grouping symbols which are also operators.

Parentheses ( ), Brackets [ ], and Braces { } – grouping only
Fraction bars – horizontal line creates numerator and denominator groups before divide
For example: $\frac{2-3}{7-4}$ means $(2-3)\div(7-4)$.
Square Roots and other radicals: The radical symbol may enclose a group, before root
For example: $\sqrt{2\cdot 3+8}$ means $\sqrt{(2\cdot 3+8)}$
Absolute values: The vertical bars may enclose a group, before absolute value
For example: $|3-17\cdot 2|$ means $|(3-17\cdot 2)|$

Step 2: **Exponents, roots, radicals.** Work from left to right.

Step 3: **Divide** and **Multiply**. Work from left to right.
Caution: Divide may come before multiply or vice-versa.

Step 4: **Subtract** and **Add**. Work from left to right.
Caution: Subtract may come before add or vice-versa.

Common Mistake: Don't combine steps. If you try to do too many steps at once, you're likely to do the steps in the wrong order. Rewrite all parts of the problem with each step of work to avoid this mistake.

Use your own paper to evaluate by hand and by calculator. You should get the same answer both ways!

1) 5^2
2) -5^4
3) $(-5)^4$
4) $(-1)^{19}$
5) $2\cdot 5^3$
6) $2^3\cdot 5$
7) $(2\cdot 5)^3$
8) 2^3-5
9) $(2-5)^3$
10) $2(-5)^3$
11) $\left(\frac{9}{10}\right)^2$
12) $\left(-\frac{4}{7}\right)^3$
13) $\left(\frac{3}{2}-\frac{5}{2}\right)\left(\frac{7}{3}+\frac{2}{3}\right)^2$
14) $\left[\frac{14}{9}\div\left(\frac{5}{3}+\frac{2}{3}\right)+\frac{8}{3}\right]\cdot\frac{1}{10}$
15) $\frac{2}{5}+\frac{4}{9}\left[-3\left(\frac{1}{5}+\frac{7}{20}\right)+\frac{3}{10}\right]$
16) $\frac{15-3^4+3^3\cdot 2}{2+2^4}$
17) $\frac{9+15\div 3\cdot 6}{6+10\cdot 0}$
18) $\frac{2\cdot 6^2+4}{3(2-6)}$
19) $\frac{1+7\div\frac{1}{5}}{-6\cdot 2+8}$
20) $\left(\frac{2^3-6}{10-2\cdot 3}\right)^2$
21) $\left|5\cdot\left(6-3^2\right)\right|$
22) $-6\left(2+\left|2\cdot 3-4^2\right|\right)$
23) $\left|3^4-9^2\right|-\left|8^2-8^3\right|$
24) $7-10^2\div 4+3\cdot 2$

① $5^2 = 5 \cdot 5 = \boxed{25}$

On scientific calculators, there is an exponent key, but it may look different on different models.

Exponent keys include

- $\boxed{\wedge}$ caret
- $\boxed{x^y}$ x to the y
- $\boxed{y^x}$ y to the x

} check your calculator to see if you type base first or the exponent first.

Most newer calculators press base first:

$\boxed{5}\ \boxed{\wedge}\ \boxed{2}\ \boxed{=}$ → result 25

$\boxed{5}\ \boxed{x^y}\ \boxed{2}\ \boxed{=}$

$\boxed{5}\ \boxed{y^x}\ \boxed{2}\ \boxed{=}$

If you get $2^5 = 32$, see next note!

Note: Some older calculators press the exponent first:

$\boxed{2}\ \boxed{x^y}\ \boxed{5}\ \boxed{=}$ 25

$\boxed{2}\ \boxed{y^x}\ \boxed{5}\ \boxed{=}$ 25

② $-5^4 = -5 \cdot 5 \cdot 5 \cdot 5$

↑ No parentheses, only one negative.

$= \boxed{-625}$

IMPORTANT: A very few calculators do this wrong! If your calc. gives +625, you'll have to be smarter than it is.

③ $(-5)^4 = (-5)(-5)(-5)(-5)$

$= +5 \cdot 5 \cdot 5 \cdot 5$

$= \boxed{625}$

On calculator, you should need ( ) to get the correct answer.

Parenthesis keys can look like this: $\boxed{(}$ $\boxed{)}$

or like this: $\boxed{\{[(\ldots}$ $\boxed{\ldots)]\}}$

④ $(-1)^{19} = \boxed{-1}$

odd exponent ⇒ odd # of negatives ⇒ negative result

(5) $2 \cdot 5^3$ order of operations: exponent 5^3 before multiply.
(NOT 2.5)

$= 2 \cdot 125$

$= \boxed{250}$

(6) $2^3 \cdot 5$ exponent before multiply

$= 8 \cdot 5$

$= \boxed{40}$

(7) $(2.5)^3$ parentheses before exp ⇒ mult first

$= 10^3$

$= \boxed{1000}$

(8) $2^3 - 5$ exponent before subtract

$= 8 - 5$

$= \boxed{3}$

(9) $(2-5)^3$ parentheses before exp ⇒ subtract first

$= (-3)^3$

$= \boxed{-27}$ odd # of negatives ⇒ neg result

(10) $2(-5)^3$ parentheses mean neg #
exp before multiply by 2

$= 2 \cdot (-125)$

$= \boxed{-250}$

(11) $\left(\frac{9}{10}\right)^2 = \frac{9}{10} \cdot \frac{9}{10}$ ← can write out exponent as repeated multiplication

$= \frac{9^2}{10^2}$ ← but it's easier to put the exp on both the numerator and denominator

$= \boxed{\frac{81}{100}}$

(12) $\left(\frac{-4}{7}\right)^3$ negative inside parentheses

$= \frac{(-4)^3}{7^3}$

$= \boxed{\frac{-64}{343}}$

(13) $\left(\frac{3}{2}-\frac{5}{2}\right)\left(\frac{7}{3}+\frac{2}{3}\right)^2$ parentheses first

$= \left(\frac{3-5}{2}\right)\left(\frac{7}{3}+\frac{2}{3}\right)^2$

$= \left(\frac{-2}{2}\right)\left(\frac{7}{3}+\frac{2}{3}\right)^2$

$= (-1)\left(\frac{7}{3}+\frac{2}{3}\right)^2$ more parentheses

$= (-1)\left(\frac{7+2}{3}\right)^2$

$= (-1)\left(\frac{9}{3}\right)^2$

$= (-1)(3)^2$ exponent before mult

$= -1 \cdot 9$

$= \boxed{-9}$

(14) $\left[\frac{14}{9} \div \left(\frac{5}{3}+\frac{2}{3}\right) + \frac{8}{3}\right] \cdot \frac{1}{10}$ parentheses first, from inside out

$= \left[\frac{14}{9} \div \left(\frac{5+2}{3}\right) + \frac{8}{3}\right] \cdot \frac{1}{10}$

$= \left[\frac{14}{9} \div \frac{7}{3} + \frac{8}{3}\right] \cdot \frac{1}{10}$ parentheses (brackets) — divide before add.

$= \left[\frac{14}{9} \cdot \frac{3}{7} + \frac{8}{3}\right] \cdot \frac{1}{10}$

$= \left[\frac{2}{9} \cdot \frac{3}{1} + \frac{8}{3}\right] \cdot \frac{1}{10}$ reduce by 7

continued

(14) continued

$= \left[\frac{2}{3}\cdot\frac{1}{1} + \frac{8}{3}\right]\cdot\frac{1}{10}$ reduce by 3

$= \left[\frac{2}{3} + \frac{8}{3}\right]\cdot\frac{1}{10}$ multiply

$= \frac{10}{3}\cdot\frac{1}{10}$ add [brackets first]

$= \boxed{\frac{1}{3}}$ reduce by 10

(15) $\frac{2}{5} + \frac{4}{9}\left[-3\left(\frac{1}{5}+\frac{7}{20}\right)+\frac{3}{10}\right]$ parentheses inside

$= \frac{2}{5} + \frac{4}{9}\left[-3\left(\frac{1\cdot4}{5\cdot4}+\frac{7}{20}\right)+\frac{3}{10}\right]$

$= \frac{2}{5} + \frac{4}{9}\left[-3\left(\frac{4+7}{20}\right)+\frac{3}{10}\right]$

$= \frac{2}{5} + \frac{4}{9}\left[-3\cdot\frac{11}{20}+\frac{3}{10}\right]$ inside brackets — mult before add.

$= \frac{2}{5} + \frac{4}{9}\left[\frac{-33}{20}+\frac{3}{10}\right]$ brackets — add

$= \frac{2}{5} + \frac{4}{9}\left[\frac{-33}{20}+\frac{3}{10}\cdot\frac{2}{2}\right]$

$= \frac{2}{5} + \frac{4}{9}\left[\frac{-33+6}{20}\right]$

$= \frac{2}{5} + \frac{4}{9}\left[\frac{-27}{20}\right]$ brackets now mean multiply

$= \frac{2}{5} + \frac{4}{9}\cdot\frac{-27}{20}$ multiply before add reduce by 9

$= \frac{2}{5} + \frac{4}{1}\cdot\frac{-3}{20}$ reduce by 4

$= \frac{2}{5} + \frac{1}{1}\cdot\frac{-3}{5}$ mult

$= \frac{2}{5} + \frac{-3}{5}$ add

$= \boxed{\frac{-1}{5}}$

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(16) $\dfrac{15-3^4+3^3\cdot 2}{2+2^4}$

$= \dfrac{(15-3^4+3^3\cdot 2)}{(2+2^4)}$

$= \dfrac{15-81+27\cdot 2}{2+16}$

$= \dfrac{15-81+54}{18}$

$= \dfrac{-66+54}{18}$

$= \dfrac{-12}{18}$

$= \boxed{\dfrac{-2}{3}}$

fraction bar creates two invisible groups — the entire numerator & entire denominator

In numerator: exp first

In denom: exp first

In numerator: mult

In denom: add

In num: subtract

REMEMBER: Add & Subtract from left to right, even if subtract is first

reduce by 6

(17) $\dfrac{9+15\div 3\cdot 6}{6+10\cdot 0}$

$= \dfrac{(9+15\div 3\cdot 6)}{(6+10\cdot 0)}$

$= \dfrac{9+5\cdot 6}{6+0}$

$= \dfrac{9+30}{6}$

$= \dfrac{39}{6}$

$= \boxed{\dfrac{13}{2}}$

Caution:

In numerator, mult and divide from left to Right means DIVIDE before add

Denom: mult before add

Numerator: mult before add

reduce by 3

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(18) $\dfrac{2 \cdot 6^2 + 4}{3(2-6)}$

numerator: exp first
denom: parentheses first

$= \dfrac{2 \cdot 36 + 4}{3(-4)}$

numerator: mult before add
denom: mult

$= \dfrac{72+4}{-12}$

$= \dfrac{76}{-12}$

Reduce by 4

$= \boxed{\dfrac{-19}{3}}$

(19) $\dfrac{1 + 7 \div \frac{1}{5}}{-6 \cdot 2 + 8}$

numerator: divide before add
denom: mult before add

$= \dfrac{1 + 7 \cdot 5}{-12 + 8}$

numerator: mult before add

$= \dfrac{1+35}{-4}$

$= \dfrac{36}{-4}$

$= \boxed{-9}$

(20) $\left(\dfrac{2^3 - 6}{10 - 2 \cdot 3}\right)^2$

parentheses first — Inside parentheses, the fraction bar creates two groups

$= \left[\dfrac{(2^3 - 6)}{(10 - 2 \cdot 3)}\right]^2$

numerator: exp before subtract
denom: mult before subtract

$= \left[\dfrac{8-6}{10-6}\right]^2$

$= \left[\dfrac{2}{4}\right]^2$

reduce by 2

continued

(20) continued

$= \left(\frac{1}{2}\right)^2$

$= \frac{1^2}{2^2}$

$= \boxed{\frac{1}{4}}$

exp on both numerator and denom.

(21) $\left| 5 \cdot (6 - 3^2) \right|$

$= \left| 5 \cdot (6 - 9) \right|$

$= \left| 5 \cdot (-3) \right|$

$= \left| -15 \right|$

$= \boxed{15}$

absolute value is a grouping symbol

exp first

parentheses before mult [or distribute]

absolute value always makes Non-Negative result → positive or zero.

(22) $-6\left(2 + \left|2 \cdot 3 - 4^2\right|\right)$

$= -6\left(2 + \left|2 \cdot 3 - 16\right|\right)$

$= -6\left(2 + \left|6 - 16\right|\right)$

$= -6\left(2 + \left|-10\right|\right)$

$= -6\left(2 + 10\right)$

$= -6\left(12\right)$

$= \boxed{-72}$

Nested Grouping Symbols from inside out

exp before mult

absolute value makes -10 positive.

parentheses before mult means add.

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(23) $|3^4 - 9^2| - |8^2 - 8^3|$

abs value are grouping symbols

exp before subtract

Note $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$
$9^2 = 9 \cdot 9 = 81$ also

$= |81 - 81| - |64 - 512|$

$= |0| - |-448|$

absolute value of zero is zero

$= 0 - |-448|$

absolute value is a grouping symbol ⇒ groups before subtract

$= 0 - 448$

$= \boxed{-448}$

could also be written as

$-|-448|$
$= -(448)$
$= \boxed{-448}$

(24) $7 - 10^2 \div 4 + 3 \cdot 2$

exp first

$= 7 - 100 \div 4 + 3 \cdot 2$

mult & div from L→R means div first

$= 7 - 25 + 3 \cdot 2$

mult next

$= 7 - 25 + 6$

Subtract and add L→R means subtract first

$= -18 + 6$

$= \boxed{-12}$